A Stackelberg Game Model for Plug-in Electric Vehicles in a Smart Grid

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What is a PEV?

- PEV = Plugin Electric Vehicle
- Motor vehicle with electricity as its source of energy
- Recharges itself from any external source of electricity
Advantages of a PEV[2]

- Have the same speed as internal combustion engines but higher torque
- Requires less maintenance
- Responsible for much less pollution
What is a Smart-Grid (SG)?

- A cyber-physical electrical grid
- Uses communication technology to gather and act on information in an automated fashion
- Optimizes the power production and distribution in the presence of renewable sources[7]
General Outline

- Motivation
- Our contributions
- Game theoretic formulation
- Existence and uniqueness of the Nash Equilibrium of the game
- Closed form solution of the game for monopolistic case and general case
- Graphical representation of the results
Motivation

In Ontario

- By 2030, all of the power generation and distribution will be controlled by SG
- By 2030 there will be large scale penetration of PEVs in the market [1]

In such a situation:

- PEVs connected to the SG compete among themselves to consume as much electrical energy as possible subject to their battery capacities
- The SG sells electricity at a particular price to PEVs with the objective to maximize its revenue without overloading the grid
- Suitable pricing strategy for PEVs connected to the SG is needed
Time of use Pricing by SG[10]

- Smart Energy Meter (SEM) is an integral part of SG
- **Time-of-use pricing:** The price is increased when the electric energy demand is high and is decreased in the opposite situation via SEM
- Such a behaviour can be modelled by *indirect penalty approach* in Game Theory[8]
Our Contributions

- Our model attempts to account for the time-of-use pricing of the SEM by using an indirect penalty approach.
- Applicable to both individual PEVs and PEV groups.
Related Work

The work in [6]

- Captures the interaction between the PEVs and the SG and the corresponding decision making process in a grid-to-vehicle scenario
- A lagrangian pricing approach is used
- An algorithm based on S-S hyperplane projection method is used [13] to solve a socially stable refinement of the proposed game
- However, the mechanism of a realistic Smart Energy Meter (SEM), that implements [10], is not considered
- Charging is assumed to be provided by charging stations, but there are places (e.g. Ontario) where charging will be done at home
The SG tries to maximize the total revenue, has a maximum energy of \( c \) for charging connected PEVs.

For the price \( p \) set by the SG, the PEVs tries to reach the Nash Equilibrium.

Such a problem can be formulated as a single leader - multiple Nash followers **Stackelberg** game.
Notations Used

$N =$ Number of different PEV models based on their battery capacities;

$b_i =$ Battery capacity of any PEV belonging to the same model number $i \in \mathcal{N}$;

$n_i =$ Number of PEVs belonging to the model number $i \in \mathcal{N}$;

$\mathcal{N} =$ Number of PEVs connected to the PEVs at a particular time;

$u_{ij} =$ Consumed energy by the $j$th PEV of model number $i \in \Omega_{ij}$;

$\bar{u}_i = \sum_{j=1}^{n_i} u_{ij} =$ Total electrical energy consumed by all the PEVs with same model $i$;
Energy Capacity Constraint: \( \bar{u} = u_i + u_{ij} + u_{ij} \leq c \)

where,
\( \bar{u} = \sum_i^N \bar{u}_i = \sum_{i=1}^N \sum_{j=1}^{n_i} u_{ij} \) = Total electrical energy consumed by all the PEVs under the SG;

\( u_i = \bar{u} - \bar{u}_i \) = Total electrical energy consumed by all PEVs under the SG except those belonging to the model number \( i \);

\( u_{ij} = \bar{u}_i - u_{ij} \) = Total electrical energy consumed by all PEVs with same model number \( i \) except the \( j \)th one;

\( \bar{\Omega} = \{ \{ u_{ij} \}_{j=1}^{n_i} \}_{i=1}^N \in \Omega : \sum_{i=1}^N \sum_{j=1}^{n_i} u_{ij} - c \leq 0 \} \) = Overall action space: compact, convex and coupled;
An example

- Only two models in the market, ⇒ $N = 2$
- Model 1 has battery capacity of 100 unit, model 2 has battery capacity of 50 unit ⇒ $b_1 = 100, b_2 = 50$
- There are 10 cars of model 1, and 5 cars of model 2 ⇒ $n_1 = 10, n_2 = 5$
- $N = 10 + 5 = 15$
- The 2nd PEV belonging to model 1 is consuming 5 unit of energy, the rest of the PEVs are consuming 65 unit of energy ⇒ $u_{12} = 5, u_{-12} = 65, \bar{u}_1 = u_{12} + u_{-12} = 70$
- All the PEVs belonging to model 2 are consuming 50 unit energy ⇒ $\bar{u}_1 = 50, \bar{u} = \bar{u}_1 + \bar{u}_2 = 70 + 50 = 120$
An Assumption

Inspired by [12], we make the following assumption:

**Assumption 1**

*Compared to the aggregate consumed energy of all the PEVs charging from the SG ($\bar{u}$), the consumed energy of any single PEV $u_{ij}(\forall j \in \mathcal{N}_i)(\forall i \in \mathcal{N})$ is so negligible that it will have no effect on the SE of the game and $u_{-i} + u_{-ij}$ can be considered equal to $\bar{u}$, i.e.,*

$$u_{-i} + u_{-ij} \approx u_{-i} + u_{-ij} + u_{ij} = \bar{u} \ (\forall j \in \mathcal{N}_i)(\forall i \in \mathcal{N})$$  (1)
Price set by SEM,

\[ p = \frac{\alpha}{c - \bar{u}} \]  

- The denominator penalizes the violation of the energy capacity constraint, so the price increases without any bound
- \( \alpha \) is a positive pricing parameter set by the SG

Pricing function of the \( j \)th PEV of model number \( i \),

\[ P_{ij} (u_{ij}, u_{-ij}, u_{-i}; \alpha) = pu_{ij} = \frac{\alpha}{c - \bar{u}}u_{ij} \]
Utility and Cost function for the PEVs

Utility function of the $j$th PEV of model number $i$,

$$U_{ij} (u_{ij}, u_{-ij}, u_{-i}; \alpha) = b_i \log (u_{ij} + 1) - s_{ij}$$

(4)

Cost function for the $j$th PEV of model number $i$ is:

$$J_{ij} (u_{ij}, u_{-ij}, u_{-i}; \alpha) = \frac{\alpha}{c - \bar{u}} u_{ij} - b_i \log (u_{ij} + 1) + s_{ij}$$

(5)
Revenue Function of the SG and the corresponding Stackelberg game

Revenue of the SG:

\[ L(p, \bar{u}) = p \bar{u} = \frac{\alpha c}{c - \bar{u}} \bar{u} \]  \hspace{1cm} (6)

The **Stackelberg game** is denoted as \( G(\{\mathcal{N} \cup \text{SG}\}, \{\{J_{ij}\}_{j=1}^{n_i}\}_{i=1}^{N}, L) \)

The **Nash followers’ (PEVs’)** game is denoted as \( G(\mathcal{N}, \hat{\Omega}_{ij}, J_{ij}; \alpha) \)
Two More Assumptions…

Assumption 2

*Strict energy capacity constraint*,

\[ \bar{u}^* = u^*_i + u^*_{ij} + u^*_{ij} < c \]  

(7)

Assumption 3

*Positivity condition at NE*,

\[ (\forall i \in \mathcal{N}) (\forall j \in \mathcal{N}_i) \quad u^*_{ij} > 0 \]  

(8)
Nash Equilibrium of the PEVs

Definition 1

Consider the Nash followers’ (PEVs’) game $G(N, \hat{\Omega}_{ij}, J_{ij}; \alpha)$, where $J_{ij}$ is given by (5). For $\forall \alpha > 0$, $\{\{u_{ij}^*\}_{i=1}^{n_i}\}_{i=1}^{N}$ is called the NE of the game if besides (7) and (8), $u_{ij}^*$ satisfies the following condition:

$$(\forall i \in N)(\forall j \in N_i)(\forall u_{ij} \in \hat{\Omega}_{ij})$$

$$J_{ij}(u_{ij}^*, u_{-ij}^*, u_{-i}^*; \alpha) \leq J_{ij}(u_{ij}, u_{-ij}^*, u_{-i}^*; \alpha)$$

(9)
Revenue maximizing condition of the SG

Definition 2

If the Nash followers’ (PEVs’) game $G(N, \hat{\Omega}_{ij}, J_{ij}; \alpha)$ achieves a unique NE as characterized by Definition 1, the leader’s (SG’s) objective is to find a pricing parameter $\alpha^* > 0$ such that it maximizes its revenue function $L$ given by (6), i.e.: 

\[(\forall \alpha > 0) \quad L(\alpha^*, \bar{u}^*) \geq L(\alpha, \bar{u}^*)\]  

(10)
Definition 3

The pair \( \{\{u_{ij}^*\}_{j=1}^{n_i}\}_{i=1}^{N}, p^* \) is called the SE of the game \( G(\{\mathcal{N} \cup SG\}, \{\{J_{ij}\}_{j=1}^{n_i}\}_{i=1}^{N}, L) \), if it satisfies (9) and (10) simultaneously.
Lemma 1

Under Assumption 1, \( J_{ij} (u_{ij}, u_{-ij}, u_{-i}; \alpha) \) in (5) can be considered equal to (approximated by) the following equivalent augmented cost function that is identical for all PEVs:

\[
J (\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_N; \alpha) = \frac{\alpha \bar{u}}{c - \bar{u}} + \sum_{r=1}^{N} \sum_{k=1}^{n_r} s_{rk} - \sum_{r=1}^{N} b_r \sum_{k=1}^{n_r} \log (u_{rk} + 1) \tag{11}
\]

and the game \( \mathcal{G}(\mathcal{N}, \hat{\Omega}_{ij}, J_{ij}; \alpha) \) is equivalent to \( \mathcal{G}(\mathcal{N}, \bar{\Omega}, J; \alpha) \).
Existence and uniqueness of the NE

Theorem 1

The PEVs’ game $G(N, \tilde{\Omega}, J; \alpha)$ admits a unique inner NE satisfying Assumptions 1, 2 and 3, if $0 < \alpha < \tilde{b}c$, where $\tilde{b} =$ weighted mean of all battery capacities $= \frac{\sum_{i=1}^{N} b_i n_i}{\sum_{i=1}^{N} n_i}$. 

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Solution for monopolistic version of the game

- Each PEV has the same battery capacity parameter $b_i = b$
- Each model type has same number of PEVs $n_i = n$, thus $N^r = Nn$
Theorem 2

The monopolistic version of the Stackelberg game $G_m(\{N \cup SG\}, \{\{J_{ij}\}_{j=1}^{n_i}\}_{i=1}^{N}, L)$ admits a unique SE given by

\[
(\{\{u_{ij}^*\}_{j=1}^{n_i}\}_{i=1}^{N}, p^*) = \left(\left\{\frac{\sqrt{cN + N^2} - N}{N}\right\}_{j=1}^{n_i}\}_{i=1}^{N}, \frac{b(\sqrt{cN + N^2} - N)}{c}\right)
\]
Solution of the game for the general case

Theorem 3

Under Assumption 1, Assumption 2 and Assumption 3, the general case of the Stackelberg game $G(\{\mathcal{N} \cup \mathcal{SG}\}, \{\{J_{ij}\}_{j=1}^{n_i}\}_{i=1}^{N}, L)$ admits a unique SE given by:

\[
(\{\{u_{ij}^*\}_{j=1}^{n_i}\}_{i=1}^{N}, p^*) = (\{\frac{b_i\sqrt{c\mathcal{N} + \mathcal{N}^2}}{\mathcal{N}\tilde{b}} - 1\}_{j=1}^{n_i}\}_{i=1}^{N}, \tilde{b}(\sqrt{c\mathcal{N} + \mathcal{N}^2} - \mathcal{N}))
\]
Survival of a PEV model in a competitive market

From the *positivity condition*:

\[ b_i > b_{th} = \frac{\tilde{b}}{\sqrt{\tilde{c} + 1}} \]  \hspace{1cm} (12)

- \( \tilde{c} = \frac{c}{N} \) = Average electrical energy per PEV supplied by the SG
- \( b_{th} = \frac{\tilde{b}}{\sqrt{\tilde{c} + 1}} \) = Threshold battery capacity
- If the battery capacity of a particular model falls below the threshold battery capacity:
  - That model can never achieve SE
  - In the long run will be out of the market
Relation between the general game and the monopolistic game

\[ \tilde{u}^* = (u_{ij}^*)_{\text{monopolistic}} \quad (\forall j \in \mathcal{N}_i)(\forall i \in \mathcal{N}) \quad (13) \]

The average of all the PEVs’ consumed energy at SE is equal to the energy consumed by a PEV at the SE in a monopolistic market.

\[ \frac{p^*}{(p^*)_{\text{monopolistic}}} = \frac{L(\alpha^*, \tilde{u}^*)}{(L(\alpha^*, \tilde{u}^*))_{\text{monopolistic}}} = \frac{\tilde{b}}{b} \quad (14) \]

In a competitive market both the price set and the revenue earned by the SG at SE increase as the ratio of \( \tilde{b} \) to \( b \) increases.
Graphical Representation of Results...

Figure 1: Change of $p^*$ with respect to $N$ for different $c$-s
Graphical Representation of Results...

Figure 2: Change of $\tilde{u}^*$ with respect to $N$ for different $c$-s

$\tilde{u}^*$ at $c=225$ MWh

$\tilde{u}^*$ at $c=350$ MWh

$\tilde{u}^*$ at $c=500$ MWh
Graphical Representation of Results...

Figure 3: Change of $b_{th}$ with respect to $N$ for different $c$-s
Conclusion

- Existence and uniqueness of Stackelberg Equilibrium is shown
- The game is solved for a monopolistic market condition
- The game for general case is solved in explicit and tractable closed form
- Condition for survival of a PEV model in a competitive market is determined
Thank You!
References

1 Towards an Ontario Action Plan For Plug-In-Electric Vehicles (PEVs), Waterloo Institute for Sustainable Energy, University of Waterloo, pp. 105-107 (2010) lagbe


3 Website of Plug’nDrive


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10 Smart Meters and Time-of-Use Prices, Website of Ontario Ministry of Energy


Implicit Programming Approach

- The Stackelberg game can be solved using *implicit programming* (IMP) approach [14] if
  - There is a one-to-one relation between $\alpha$ and $u_{ij}^* \ (\forall j \in \mathcal{N}_i)(\forall i \in \mathcal{N})$
  - $\alpha$ is Lipschitz continuous and directionally differentiable in $u_{ij}^* \ (\forall j \in \mathcal{N}_i)(\forall i \in \mathcal{N})$
- Conditions are satisfied, so IMP can be used
- Steps in IMP:
  - In the SG’s revenue $L$, substitute the value of $\alpha$ in terms of $\tilde{u}^*$
  - Maximize the resultant function with respect to $\tilde{u}^*$
  - Substitute the value of the resultant maximizer $\tilde{u}^*$ in $\alpha$ and thus obtain the SE.